Problem Solving

Terminology

- Problem solving
  - Goal is not readily available
  - Use knowledge to reach goal
- Initial state
  - State you are in when you encounter problem

Terminology: Problem Space

- Set of choices
  - Found at each step of solving the problem
- Includes
  - Initial, intermediate, and goal states
  - Knowledge being applied to problem
  - Knowledge that could be applied
  - External devices, objects, resources
- Range on continuum from large to small
Terminology: Operators

- Legal operators or moves performed during problem solving
- Classes
  - Algorithms
  - Heuristics

Algorithm

- Precise rule
  - Always yields a correct solution to the problem
- Frequently slow and inefficient
  - Exhaustive search
    - Search entire problem space
    - Difficult for humans
    - Easy for computers

Heuristic

- Rule of thumb
  - Likely, but not guaranteed, to generate solution
- Selective search of problem space
  - Examine only those parts likely to lead to solution
- Example – multiple choice tests
  - “B” or “C” answers the most frequent
Terminology: Goal state

- Ultimate solution to problem
- Problem defined in terms of goal state specification
  - Well-defined problems
  - Ill-defined problems

Well-defined and Ill-defined Problems

- Well-defined
  - Clear goal
  - Small set of information to start
  - Guidelines or rules
- Ill-defined
  - Unclear goal
  - Starting information, operators or both are vaguely specified
  - Many real world problems

Types of Problems

- Problems of arrangement
- Problems of inducing structure
- Problems of transformation
- Insight problems
Problems of Arrangement

• Present objects
  – Require problem solver to arrange objects
  • Satisfy criterion
  – Only a few arrangements result in solution
• Anagrams

Arrangement Problem Skills

• Fluency in generating possibilities
  – Generate many potential solutions
  – Discard inappropriate solutions
• Retrieval of solution patterns
  – Anagram example – words from memory
• Knowledge of constraining principles
  – Anagram example – relative frequency of words in the language

Problems of Inducing Structure

• Finding a pattern among a fixed set of relations
  – Some objects given
    • Figure out how objects relate
• Examples
  – Series extrapolation
  – Analogy
  – Progressive matrices
Inducing Structure Skills

• Identifying relationships among components
• Fitting relationships into patterns

Problems of Transformation

• Changing initial state until it matches goal state
• Supply goal state
• Example
  – Book burners and book lovers problem
• Skills
  – Planning

Insight Problems

• Insight problems (Tower problem)
  – Solution suddenly enters the mind
    • Immediately recognized as correct
    • Restructure conceptualization of problem
• Non-insight problems (anagram problem)
  – Solve problem gradually
  – Reasoning skills and routine procedures
• Solution accompanied by an “AHA!” experience
Solutions to insight problems

- Metcalf & Wiebe (1987) studied insight problems
- Compared insight to algebra problem solving
  - “Warmth ratings”

Insight Controversy

- Some researchers question the concept of insight
- Propose
  - Gradually work towards a solution
  - Solution just “feels” different
- Some non-insight problems solved by restructuring

Approaches to Studying Problem Solving

- Classic approach
- Computer simulation
- Verbal Protocols
Classic Problem Solving Research

- Gestalt psychologists
  - Perception and the structure of patterns
  - Arrangement problems
- Kohler (1925) & Sultan
  - Cage contained sticks and boxes
  - Rearrange objects to get fruit

Computer Simulation

- Develop computer programs for problem solving
  - Based on human problem solving performance
- Examine
  - Sequence of steps
  - Constraints
  - Representation

Computer Simulation

- Computer model created
  - Based on theory of human problem solving performance
- Computer modeling
  - Mimic human performance
    - Good understanding of problem solving performance
  - Fails to accurately model performance
    - Theory needs to be revised
Advantages of Computer Simulation

• Forces researchers to be explicit & specific
  – No generalizations
  – Pay attention to details
• Can be used to make predictions
  – Tested on human subjects

Disadvantages of Computer Simulation

• “Just get the program running”
  – No longer mimics human performance
• Forced to work on artificial or very simple problems
  – Examine limited and artificial types of problem solving

Real Problems

• Traveling Salesperson Problem
  – Each city in a network of connected cities must be visited exactly once.
  – The goal is to find the shortest trip
• Medical diagnoses
Verbal Protocols

- “Think out loud” data
- Provides information
  - Sequence of steps taken during problem solving
  - Constraints observed
  - Heuristics used

Verbal Protocols (cont.)

- Messy/Noisy data
  - Great deal of information
  - Very rich data set
- Problem solving graphs
  - Transcribed, transformed, and reduced protocols
  - Trace mental representation of problem
  - Show possible states and operators
    - Applied to knowledge in each state
    - Change the state

Problem Representation

- Symbols
- Lists
- Matrices
- Hierarchical Tree
  Diagram
- Graphs
- Visual Images
Symbols

Use symbols to represent variables in problem
– Algebra

Disadvantages
– Difficult to use with ill-defined problems
– Difficult to translate words into symbols

Matrices

– Chart
– Shows possible combinations of problem
– Usefulness
  – Complex problems with categorical information
Hierarchical Tree Diagram

- Uses tree-like structure
- Specifies every possible outcome
- Useful for assessing probability of outcomes

Hierarchical Tree Diagram. Suppose that you are playing a coin game with two children, Chris and Pat. You toss each of three coins. If it's heads, the coin goes to Chris; if it's tails, it goes to Pat. What is the probability that one child will get to keep all three coins? Solving this problem with a list format typically leads to an incorrect outcome. (Kersten, 1984)

Visual Images

- Images are free from rationality
  - Escape from boundaries of traditional representations
- Images are concrete
  - Can serve as symbols for more abstract concepts
Problem Solving Heuristics

- Hill climbing/ Simple search
- Means-ends
- Working backwards
- Analogy

Simple Search / Hill Climbing Heuristic

- Examine all possible operators to identify all possible next states
- Compare each of these states to the goal state
- Choose the state that is closest to the goal state

Local maxima

- Local maxima can cause hill-climbing heuristic to fail
- Consider only a limited range of choices
  - Look one move ahead
- Many solutions require moving away from a goal to ultimately achieve it
Means-Ends Heuristic
(Forming Subgoals)

- Divide the problem into smaller problems
- Solve smaller problems
- Means – ends
  - Figure out ends for each sub problem and the means by which those ends will be achieved
- Sometimes solving subgoals requires the formation of other subgoals

Main Problem: Getting from New York to San Francisco
- Subgoal 1: Get on plane to San Francisco
  - Subgoal 1a: Get to airport
    - Solution: Use map to drive to airport
  - Subgoal 1b: Get on plane
    - Sub-subgoal: Find plane
      » Solution: Consult arrival/departure boards

Working Backwards

- Start from goal
- Determine what needs to be done for goal to be true
- Repeat at each step
- Useful
  - # states preceding goal is small
  - # states following start is large
Analogy

- Using a solution to an earlier problem to help solve a new one
  - Very common method of problem solving
- Target domain
  - The problem you are trying to solve
- Source domain
  - The problem or domain you use as an analogy to solve the current problem

Problem Structure

- Require us to ignore the surface details of a problem
- Need to determine problem structure
- Problem isomorphs
  - Same underlying structures and solutions
  - Different surface details

Retrieving Correct Source

- The use of analogy requires application of appropriate source analog
- Sometimes difficult to retrieve appropriate analog from memory
- Gick & Holyoak (1980)
  - Examined the use of analogies in problem solving in two studies
Gick & Holyoak (1980) Study 1

- Parade problem and then Radiation problem
  - 49% solution rate
- Attacking General problem and then Radiation problem
  - 76% solution rate
- Radiation problem alone
  - 8% solution rate

Gick & Holyoak (1980) Study 2

- Read the Attacking General problem and its solution and then solve the Radiation problem
- Hint
  - 92% solution rate
- No hint
  - 20% solution rate

Creation of a general schema

- Schema creation aids in solving problem isomorphs
- Gick & Holyoak (1983)
  - Formation of convergence schema requires that people compare two analogous stories
  - Makes them think about the solution in general terms
Barriers to Problem Solving

- Problem solving involves some sort of obstacle to overcome in the process of reaching a goal
- Fixation
  - An impediment to problem solving
  - When fixated, you are stuck on some aspect of a problem which blocks your ability to reach a solution

Perceptual fixation

- Involves perceptual (visual/spatial) assumptions about the problem domain that blocks your ability to reach a solution
Mental Set

- Rule-based fixation
- Getting stuck on a set of rules to solve a problem

<table>
<thead>
<tr>
<th>Problem</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Measure set this much more</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>117</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>46</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>61</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>61</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>62</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>49</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>19</td>
<td>10</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>